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APPLYING THE EXPLOSION ANALOGY TO THE CALCULATION OF HYPERSONIC FLOWS

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After Tsien [1], Hayes [2] and Il'iushin [3] had established the analogy between hypersonic flow past slender bodies,and unsteady flows in a space with one fewer dimensions, many researchers sought to establish which steady flow corresponds to the motion of a gas produced by an intense explosion. The authors of the earliest studies [4–8] assumed that the gas particles in an explosion of a flat or filament charge move in the same way as in flow near a blunt plate or semi-infinite cylinder at a zero angle of attack relative to the free stream. The thickness of the streamlined bodies were assumed to be infinitesimal; the bluntness of their leading edges was taken to be the direct analog of the action of a concentrated force on the ambient medium. The resulting analogy made it possible to isolate the most salient common features of the two effects, but suffered from one drawback: the density at the plate and cylinder surfaces turned out to equal zero, and the entropy to be infinite.

Cheng [9], Sychev [10, 11] and Yakura [12] subsequently developed the notion of a high-entropy layer whereby the thickness of streamlined bodies increases to infinity downstream, while the entropy remains finite over the entire contour. They emphasized that flow in a high-entropy layer differs from that in the rest of space, and that the use of the hypothesis of plane cross sections to calculate this layer entails considerable errors.

The results of Sychev [10, 11] and Yakura [12] are thoroughly analyzed below. It is shown that these results are obtainable directly from the theory of intense explosions as developed by Sedov [13, 14] and Taylor [15]. This possibility means that the analogy between unsteady flows and hypersonic flow past slender bodies is valid in the first approximation throughout the domain beyond the front of the bow shock wave. This includes the domain adjacent to the contour of the body. The contour itself can be determined simply by choosing an appropriate value of the entropy at the particle trajectory which generates it; the equation of the trajectory can be found by solving the explosion problems in Lagrange variables [16].

1. We assume that the motion of the gas is axially symmetric. Our principal conclusions will be equally valid for plane-parallel flows, however. We denote the axes of the cylindrical coordinate system by x and r, directing the x-axis along the velocity vector of the unperturbed stream. Following [10–12], we shall consider the inverse problem, i. e. we shall prescribe the form of the shock wave $r = r_s(x)$, and determine the contour of the streamlined body in the course of solution. Using the explosion analogy to calculate the hypersonic flow, we set

$$r_s = C\sqrt{x} \tag{1.1}$$

where C is an arbitrary constant.

The principal contribution made by Sychev [10] was to determine the shape of the body far away from the point of intersection of the central streamline and the shock front. His expression for the contour $r = r_b(x)$ of the body is

$$r_b = Cx^{1/2}\left\{1 + \frac{\varkappa - 1}{\varkappa + 1}\int_1^0 G(x, \eta)\left[1 - \frac{\varkappa C^2}{(\varkappa + 1)^2}\frac{G(x, \eta) H(\eta)}{x}\right]^{-1/2} d\eta\right\}^{1/2} \tag{1.2}$$

Here $\varkappa$ is the Poisson adiabatic exponent, the function H is equal to the ratio of the pressure in the perturbed flow domain to the pressure beyond the shock wave, and the quantity

$$G = \left[\frac{x}{H}\left(x\eta_x' + \frac{C^2}{4}\right)^{-1}\right]^{1/\varkappa} \tag{1.3}$$

Formula (1.2) is not valid for small x, since the perturbations of the velocity field turn out to be finite and cannot be described by a theory based on the unsteady flow analogy. Conversely, this formula becomes more precise the larger the x-coordinate. It is therefore expedient to simplify it by taking the limit and letting $x \to \infty$.

To do this we make use of the integration variable η and the self-similar variable λ introduced by Sedov [16]. Denoting the ratio of the velocity in the perturbed flow zone to the velocity beyond the shock front by f, we obtain [10]

$$\eta = \exp\left(2\int_1^\lambda \left(\lambda - \frac{2}{\varkappa + 1} f\right)^{-1} d\lambda\right)$$

Let us also introduce the function g defined as the ratio of the density at an arbitrary point lying between the shock wave and the body to the density due to intense shock compression of the gas. The functions f and g and their first derivatives are related by the expression

$$\frac{1}{g}\frac{dg}{dx} + \left(f - \frac{\varkappa+1}{2}\lambda\right)^{-1}\left[\left(\frac{df}{d\lambda} - \frac{\varkappa+1}{2}\right) + \left(\frac{f}{\lambda} + \frac{\varkappa+1}{2}\right)\right] = 0$$

which appears in monograph [17]. This relation is readily transformable into

$$\frac{d}{d\lambda}\ln\left[\lambda g\left(\frac{\varkappa+1}{2}\lambda - f\right)\right] = 2\left(\lambda - \frac{2}{\varkappa+1} f\right)^{-1}$$

from which we infer that

$$\eta = \frac{2}{\varkappa - 1}\lambda g\left(\frac{\varkappa+1}{2}\lambda - f\right) \tag{1.4}$$

By definition, $H(\eta) = h(\lambda)$. Making use of the Eq. (1.4), we can rewrite formula (1.3) as

$$G = x^{1/\varkappa}\left\{h\left[\frac{2}{\varkappa-1}\lambda g\left(\frac{\varkappa+1}{2}\lambda - f\right)x + \frac{C^2}{4}\right]\right\}^{-1/\varkappa}$$

The expression in square brackets in the right side of the latter equation can be simplified with the aid of the adiabatic criterion [17]

$$\frac{2}{\varkappa-1}\lambda g h\left(\frac{\varkappa+1}{2}\lambda - f\right) = g^\varkappa$$

The function G now assumes the final form

$$G = \left(g^\varkappa + \frac{C^2}{4}\frac{h}{x}\right)^{-1/\varkappa} \tag{1.5}$$

Converting from the variable η to the self-similar variable λ in Eq. (1.2) for the contour of the required body, we obtain

$$r_b = Cx^{1/2}\left\{1 + 2\int_1^0 G(x, \lambda)\left[1 - \frac{\varkappa C^2}{(\varkappa+1)^2}\frac{G(x, \lambda)h(\lambda)}{x}\right]^{-1/2}\lambda g(\lambda)\, d\lambda\right\}^{1/2} \tag{1.6}$$

The function $G \to g^{-1}$ as $x \to \infty$ for finite values of λ. As $\lambda \to 0$ the ratio $g \to 0$ and $h \to h_0 \neq 0$. This follows from Sedov's asymptotic formulas [16]. Hence, as $\lambda \to 0$ and $x \to \infty$, the second of the two terms in parentheses in the right side of Eq. (1. 5) may turn out to be larger than the first, and for $\lambda = 0$ we have

$$G = \left(\frac{4}{C^2 h_0}\right)^{1/\varkappa} x^{1/\varkappa}$$

This implies that the ratio $G / x \to 0$ as $x \to \infty$ for all values of λ. Making use of this fact, we can write the expansion

$$\left[1 - \frac{\varkappa C^2}{(\varkappa+1)^2}\frac{Gh}{x}\right]^{-1/2} = 1 + \frac{\varkappa C^2}{2(\varkappa+1)^2}\frac{Gh}{x} + \ldots \tag{1.7}$$

In order to determine the asymptotic behavior of the contour generatrix of the required body for large values of the x-coordinate, we must use only the first term of series (1. 7) in computing the integral in the right side of (1. 6). It is not difficult to show that the remaining terms of this series make a contribution of lower order in x to this integral. This means that in the first approximation

$$r_b = Cx^{1/2}\left\{1 + 2\int_1^0 \left(g^\varkappa + \frac{C^2}{4}\frac{h}{x}\right)^{-1/\varkappa}\lambda g\, d\lambda\right\}^{1/2} \tag{1.8}$$

Expansion of the integrand of the above expression in a series is no longer possible for large x. We therefore have

$$\int_0^1 \left(g^\varkappa + \frac{C^2}{4}\frac{h}{x}\right)^{-1/\varkappa}\lambda g\, d\lambda = \left(\int_0^\varepsilon + \int_\varepsilon^1\right)\left[\left(g^\varkappa + \frac{C^2}{4}\frac{h}{x}\right)^{-1/\varkappa}\lambda g\, d\lambda\right] = J_1 + J_2$$

where the parameter ε must be chosen in such a way that, on the one hand,

$$g^\varkappa(\varepsilon) \gg \frac{C^2}{4}\frac{h(\varepsilon)}{x} \tag{1.9}$$

and on the other $\varepsilon \ll 1$. Series expansion of the integrand in J_2 is possible by virtue of condition (1. 9). Making use of this condition, we obtain

$$J_2 = -\frac{1}{2} + \frac{\varepsilon^2}{2} - \frac{C^2}{4\varkappa}\frac{1}{x}\int_1^\varepsilon \frac{h}{g^\varkappa}\, d\lambda + \ldots$$

To compute the integral J_1 we first transform the expression

$$\left(g^\varkappa + \frac{C^2}{4}\frac{h}{x}\right)^{-\frac{1}{\varkappa}} = g_0 \lambda^{\frac{2}{\varkappa-1}} g^{-1} \mu^{-\frac{1}{\varkappa}}(1+m)^{-\frac{1}{\varkappa}}$$

$$m = \frac{C^2}{\mu g^\varkappa x}\left(g_0^\varkappa \lambda^{\frac{2\varkappa}{\varkappa-1}} h - g^\varkappa h_0\right), \quad \mu = g_0^\varkappa \lambda^{\frac{2\varkappa}{\varkappa-1}} + \frac{C^2}{4}\frac{h_0}{x} \tag{1.10}$$

where the constants g_0 and h_0 are the coefficients of the first terms of the asymptotic expansions of the functions

$$g = \lambda^{\frac{2}{\varkappa-1}}\left(g_0 + g_1 \lambda^{\frac{2\varkappa}{\varkappa-1}} + \ldots\right), \quad h = h_0 + h_1 \lambda^{\frac{2\varkappa}{\varkappa-1}} + \ldots \tag{1.11}$$

for small λ. Making use of asymptotic expressions (1. 11), we can readily show that for large x and $0 \leqslant \lambda \leqslant \varepsilon$ the quantity $m \ll 1$.

Bearing this inequality in mind, we find that in the first approximation

$$J_1 = g_0 \int_0^{\varepsilon} \lambda^{\frac{\varkappa+1}{\varkappa-1}} \mu^{-\frac{1}{\varkappa}} \, d\lambda$$

Converting from integration over λ to integration over μ in accordance with formula (1.10), we can compute the value of J_1 in finite form. Retaining only the principal terms, we obtain

$$J_1 = -\frac{\varepsilon^2}{2} + \frac{1}{2} 2^{-\frac{2(\varkappa-1)}{\varkappa}} g_0^{1-\varkappa} h_0^{\frac{\varkappa-1}{\varkappa}} C^{\frac{2(\varkappa-1)}{\varkappa}} x^{-\frac{\varkappa-1}{\varkappa}} + \ldots$$

Now let us collect the above results and substitute them into Eq. (1.8). In the final analysis the behavior of the generatrix of the streamlined body contour as $x \to \infty$ is given by the expression

$$r_b = 2^{-\frac{\varkappa-1}{\varkappa}} g_0^{\frac{1-\varkappa}{2}} h_0^{\frac{\varkappa-1}{2\varkappa}} C^{\frac{2x-1}{\varkappa}} x^{\frac{1}{2\varkappa}} \tag{1.12}$$

In comparing formula (1.12) with the analogous formula of Yakura [12] it is convenient to convert to dimensionless variables by dividing the coordinates by the radius r_* of the shock front at its point of intersection with the axis of symmetry. From Eq. (1.1) defining the form of the shock wave we obtain

$$r_s/r_* = \sqrt{2x/r_*} \quad (C = \sqrt{2r_*}) \tag{1.13}$$

Finally, recalling the equations [16, 17]

$$g_0 = 2^{-\frac{2\varkappa}{(\varkappa-1)(2-\varkappa)}} \varkappa^{\frac{3\varkappa-4}{(\varkappa-1)(2-\varkappa)}} (\varkappa+1)^{\frac{2}{\varkappa-1}}, \quad h_0 = 2^{-\frac{2}{2-\varkappa}} \varkappa^{\frac{2(\varkappa-1)}{2-\varkappa}} (\varkappa+1) \tag{1.14}$$

for the coefficients g_0 and h_0, we can rewrite formula (1.12) as

$$\frac{r_b}{r_*} = 2^{\frac{4-\varkappa}{2\varkappa(2-\varkappa)}} \varkappa^{\frac{2-\varkappa^2}{2\varkappa(2-\varkappa)}} (\varkappa+1)^{-\frac{\varkappa+1}{2\varkappa}} \left(\frac{x}{r_*}\right)^{\frac{1}{2\varkappa}} \tag{1.15}$$

2. Now let us turn to Yakura's paper [12]. Yakura found the shape of the body corresponding to shock wave (1.13) by constructing the solution of the gas dynamics equations by the well-developed method of combining exterior and interior asymptotic expansions (the principles of this method are presented in detail by Van Dyke [18]). He assumed that the exterior flow region was described by the solution of the intense-blast problem obtained by Sedov [13, 14] and Taylor [15]; the interior expansion gave him the velocity field in the high-entropy layer adjacent to the streamlined body. Yakura also assumed that the perturbation theory [1–3] based on the hypothesis of plane cross sections by analogy with unsteady flows is not directly applicable to the study of flows in high-entropy layers.

We begin the analysis of the interior expansion formulas [12] with the equation

$$\frac{r_b}{r_*} = \left(\frac{\varkappa}{\varkappa+1}\right)^{1/2} \left(\frac{2}{h_0}\right)^{\frac{1}{2\varkappa}} \left(\frac{x}{r_*}\right)^{\frac{1}{2\varkappa}} \tag{2.1}$$

of the contour generatrix of the required body. It is easy to show with the aid of the second equation of (1.14) that the above expression is identical to Eq. (1.15), which follows from relation (2.1). Thus, the shape of the streamlined body corresponding to shock wave (1.13) as obtained by Yakura and Sychev turns out to be the same in the first approximation, even though the methods of investigating the problem which underlie their studies are quite different. This explains the good qualitative agreement of the results obtained by direct computation of the integral appearing in (1.2) with the results which follow from formula (2.1). The only disparity is due to the fact that the values of x/r_* chosen in [10, 12] were not too large. No deeper reasons for the disparity exist [19].

Yakura's interior expansion formulas [12] make it possible to determine not only the contour of the streamlined body, but also the structure of the high-entropy layer adjacent to it. The independent variables in these expansions are the longitudinal coordinate x and the stream function ψ; the transverse coordinate r is specified by way of the equation

$$\frac{r}{r_*} = \left(\frac{\varkappa}{\varkappa+1}\right)^{1/2} \left(\frac{2}{h_0}\right)^{\frac{1}{2\varkappa}} \left(\frac{2\psi}{\rho_\infty V_\infty r_*^2}+1\right)^{\frac{\varkappa-1}{2\varkappa}} \left(\frac{x}{r_*}\right)^{\frac{1}{2\varkappa}} \tag{2.2}$$

Here ρ_∞ and v_∞ denote the density and velocity in the free stream. For $\Psi = 0$ relation (2.2) implies (2.1). Making use of this equation, we can express the stream function in terms of x and r and then obtain explicit expressions for the transverse component v_r of the particle velocity, the pressure p, and the density ρ as functions of the cylindrical coordinates. Replacing the coefficient h_0 by its expression as given in (1.14), we find on the basis of [12] that

$$\frac{v_r}{V_\infty} = \frac{1}{2\varkappa}\left(\frac{x}{r_*}\right)^{-1}\frac{r}{r_*}, \qquad \frac{p}{\rho_\infty V_\infty^2} = 2^{-\frac{2}{2-\varkappa}} \varkappa^{\frac{2(\varkappa-1)}{2-\varkappa}} \left(\frac{x}{r_*}\right)^{-1} \tag{2.3}$$

$$\frac{\rho}{\rho_\infty} = 2^{-\frac{4-\varkappa}{(\varkappa-1)(2-\varkappa)}} \varkappa^{\frac{3\varkappa-4}{(\varkappa-1)(2-\varkappa)}} (\varkappa-1)^{-1}(\varkappa+1)^{\frac{\varkappa+1}{\varkappa-1}} \left(\frac{x}{r_*}\right)^{-\frac{1}{\varkappa-1}} \left(\frac{r}{r_*}\right)^{\frac{2}{\varkappa-1}}$$

As regards the longitudinal component v_x of the particle velocity, its deviation from the velocity of the free stream is

$$\frac{v_x}{V_\infty} - 1 = -2^{\frac{3}{\varkappa-1}} \varkappa^{\frac{2-\varkappa}{\varkappa-1}} (\varkappa+1)^{-\frac{\varkappa+1}{\varkappa-1}} \left(\frac{x}{r_*}\right)^{\frac{2-\varkappa}{\varkappa-1}} \left(\frac{r}{r_*}\right)^{-\frac{2}{\varkappa-1}} \tag{2.4}$$

We can also write out the expression for the entropy,

$$\frac{p/(\rho_\infty V_\infty^2)}{(\rho/\rho_\infty)^\varkappa} = 2(\varkappa-1)^\varkappa(\varkappa+1)^{-(\varkappa+1)} \frac{1}{2\Psi/(\rho_\infty U_\infty r_*^2)+1} \tag{2.5}$$

which naturally depends on the stream function Ψ alone. The maximum value of the entropy corresponds to $\Psi = 0$, i.e. to compression of the gas at the normal shock wave.

3. Now let us consider some relations from the intense-explosion theory developed by Sedov [13, 14] and Taylor [15]. We denote the time by t; the quantity E is proportional to the energy released upon detonation of a filament charge of unit length. The coordinate of the shock wave is then given by

$$r_s = \left(\frac{E}{\rho_\infty}\right)^{1/4} \sqrt{t} \tag{3.1}$$

In using the analogy to calculate hypersonic flows the quantity E is identified with the constant F_x proportional to the force; the time t is related to the x-coordinate by the expression [4–8]

$$t = x \,/\, U_\infty \tag{3.2}$$

Its substitution into formula (3.1) yields

$$\frac{r_s}{r_*} = C_{x1}^{1/4}\left(\frac{x}{r_*}\right)^{1/2} \qquad \left(C_{x1} = \frac{F_x}{\rho_\infty U_\infty^2 r_*^2}\right) \tag{3.3}$$

In order for Eq. (3.3) to coincide with (1.13), we must set the drag coefficient $C_{x_1} = 4$. We assume from now on that this condition is fulfilled. Sedov [16] showed that the following asymptotic expansions are valid near the blast center:

$$v_r = \frac{1}{2\varkappa}\frac{r}{t}, \quad p = k_2\rho_\infty\left(\frac{E}{\rho_\infty}\right)^{1/2}\frac{1}{t}, \quad \rho = k_1\rho_\infty\left(\frac{E}{\rho_\infty}\right)^{-\frac{1}{2(\varkappa-1)}} t^{-\frac{1}{\varkappa-1}} r^{\frac{2}{\varkappa-1}} \qquad (3.4)$$

The coefficients k_1 and k_2 in these expressions are related to the Poisson adiabatic coefficient in the following way:

$$k_1 = 2^{-\frac{2}{(\varkappa-1)(2-\varkappa)}}\varkappa^{\frac{3\varkappa-4}{(\varkappa-1)(2-\varkappa)}}(\varkappa-1)^{-1}(\varkappa+1)^{\frac{\varkappa+1}{\varkappa-1}}, \quad k_2 = 2^{-\frac{4-\varkappa}{2-\varkappa}}\varkappa^{\frac{2(\varkappa-1)}{2-\varkappa}}$$

Converting from the time t to the coordinate x in accordance with (3.2) in expansions (3.4) and recalling the two above expressions, we see that the indicated expansions coincide precisely with Yakura's formulas (2.3). This coincidence implies the validity of the hypothesis of plane cross sections [1–3] in the case of the high-entropy layer adjacent to the surface of the streamlined body. In fact, Yakura's interior expansion [12] actually represents the asymptotic form of the solution of the intense-explosion problem for $r \to 0$; it is this asymptotic form which he matched with the complete solution of the same problem. In other words, the analogy between unsteady flows and hypersonic flow past slender bodies can be used to compute the entire domain situated between the front of the shock wave and the surface of the body.

We must now consider the shape of the body itself. As we have seen, its contour must be generated by the trajectory of one of the particles set in motion by the detonation wave. In order to see this let us make use of the solution of the blast problem in Lagrange variables as it appears in Sedov's monograph [16]. This solution is given in parametric form, the parameter being the dimensionless velocity $V = tv_r/r$. The axis of symmetry of the flow corresponds to the value $V = 1/(2\varkappa)$. Let us denote by r_0 the initial coordinate of the particle prior to the passage of the shock wave, and set

$$V = \frac{1}{2\varkappa}(1 + \Delta)$$

It is easy to show that for small Δ the solution [16] of the intense-explosion problem in Lagrange variables has the following asymptotic form:

$$\frac{r}{r_s} = 2\varkappa^{1/2}(\varkappa-1)^{-\frac{\varkappa-1}{2\varkappa}}(\varkappa+1)^{-\frac{\varkappa+1}{2\varkappa}}\Delta^{\frac{\varkappa-1}{2\varkappa}}$$

$$\frac{r_0}{r_s} = 2^{-\frac{\varkappa-1}{2-\varkappa}}\varkappa^{\frac{1}{2-\varkappa}}(\varkappa-1)^{-1/2}\Delta^{1/2}$$

Eliminating the parameter Δ and making use of expression (3.3) for the coordinate r_s of the shock wave, we obtain

$$\frac{r}{r_*} = 2^{\frac{4-\varkappa}{2\varkappa(2-\varkappa)}}\varkappa^{\frac{2-\varkappa^2}{2\varkappa(2-\varkappa)}}(\varkappa+1)^{-\frac{\varkappa+1}{2\varkappa}}\left(\frac{r_0}{r_*}\right)^{\frac{\varkappa-1}{\varkappa}}\left(\frac{x}{r_*}\right)^{\frac{1}{2\varkappa}} \qquad (3.5)$$

For $r_0 = r_*$ formula (3.5) coincides completely with (1.15), which implies that the contour of the streamlined body is formed by the trajectory of a particle propelled by the detonation wave. The resulting value of the coordinate r_0 is related to the appropriate choice of the entropy along the trajectory (contour). In fact, in the intense-explosion problem [16] we have

$$\frac{p}{\rho^\varkappa} = \frac{1}{2}(\varkappa-1)^\varkappa(\varkappa+1)^{-(\varkappa+1)}\frac{E}{\rho_\infty{}^\varkappa}\frac{1}{r_0{}^2} \qquad (3.6)$$

Converting to dimensionless variables, we obtain

$$\frac{p/(\rho_\infty U_\infty^2)}{(\rho/\rho_\infty)^\varkappa} = 2(\varkappa-1)^\varkappa(\varkappa+1)^{-(\varkappa+1)}\left(\frac{r_*}{r_0}\right)^2 \tag{3.7}$$

Let us compare the above value of the entropy with that given by formula (2.5) in which the streamlined body is associated with the value $\Psi = 0$. The two values turn out to be equal for $r_0 = r_*$. Thus, direct application of explosion theory to the determination of the streamlined body contour requires only the correct specification of the entropy at the particle trajectory which is its generatrix. This value of the entropy occurs with shock compression of the gas at the normal shock wave front in a hypersonic stream. It is the maximum permissible entropy, since the entropy beyond an oblique shock wave must be smaller. On the other hand, according to the solution of the intense-explosion problem the entropy in the particles can increase without limit with decreasing distance to the axis of symmetry. The maximum permissible entropy is

$$\left[\frac{p/(\rho_\infty U_\infty^2)}{(\rho/\rho_\infty)^\varkappa}\right]_{\max} = 2(\varkappa-1)^\varkappa(\varkappa+1)^{-(\varkappa+1)}$$

which agrees with relation (2.5). This value defines that region of unsteady flow which can be used to compute the hypersonic stream. In the remaining part of the unsteady flow resulting from an intense explosion of a filament charge the entropy values in the particles exceed those attainable in a hypersonic steady stream.

The streamlines near the body contour are normalized by the expression

$$\frac{2\Psi}{\rho_\infty U_\infty r_*^2} = \left(\frac{r_0}{r_*}\right)^2 - 1 \tag{3.8}$$

which follows from a comparison of formulas (2.5) and (3.7). On fulfilment of condition (3.8), Eq. (3.5) for the trajectory of an arbitrary particle becomes expression (2.2) which occurs in Yakura's interior expansion.

Correction (2.4) for the longitudinal component of the velocity vector is equally easy to find on the basis of intense-explosion theory. To this end we need merely substitute formulas (3.4) reduced to the form (2.3) into the Bernoulli integral,

$$\frac{1}{2}(v_x^2 + v_r^2) + \frac{\varkappa}{\varkappa-1}\frac{p}{\rho} = \frac{1}{2}U_\infty^2$$

According to small-perturbation theory a particle trajectory must be described by the solution of the ordinary differential equation

$$\frac{dr}{dt} = v_r(t, r)|_{r\to 0} = \frac{1}{2\varkappa}\frac{r}{t}$$

Integrating this equation, we obtain

$$r = At^{\frac{1}{2\varkappa}} \tag{3.9}$$

In order to determine the arbitrary constant A, we substitute asymptotic expansions (3.4) for the pressure and density into the left side of Eq. (3.6). Allowance for relation (3.9) between the cylindrical coordinate and the time enables us to find A from the initial position r_0 of the particle. It is easy to show that on converting from t to x according to (3.2) we again obtain formula (3.5). Thus, the condition of entropy conservation in a particle enables us to find the correct value of the constant in the asymptotic expansion of its trajectory as $t \to \infty$.

4. Finally, let us consider the results of [11], where Sychev again posed the problem of finding the shape of a body generating a shock wave of the form (1.1) and (1.13) in a steady hypersonic stream. He solved the problem by the method of deformed Poincaré-Lighthill-Ho coordinates described in [18]. His scale of reference for the cylindrical coordinates was the diameter d of the blunt nose of the body. Sychev wrote the expression of the shock front in the form

$$r_s/d = \varkappa_1 C_{x_2}^{1/4} \quad (x/d)^{1/2}$$

This formula becomes identical to formula (1.13) if we set

$$\varkappa_1^4 C_{x2} = C_{x1} = 4 \tag{4.1}$$

The scale factor d then becomes identical to the radius of curvature r_* of the shock wave at its point of intersection with the axis of symmetry of the stream. According to [11], the required contour equation can be written as

$$\frac{r_b}{d} = 2^{-\frac{\varkappa-1}{\varkappa}} \varkappa^{\frac{1}{2}} (\varkappa+1)^{-\frac{\varkappa+1}{2\varkappa}} \varkappa_1^2 \varkappa_2^{-\frac{1}{2\varkappa}} C_{x2}^{\frac{2\varkappa-1}{4\varkappa}} \left(\frac{x}{d}\right)^{\frac{1}{2\varkappa}}$$

where the constant $\varkappa_2$ can be expressed in terms of $\varkappa_1$ and the coefficient h_0 introduced above by means of the formula

$$\varkappa_x = \frac{h_0 \varkappa_1^2}{\varkappa+1}$$

Making use of this equation, we immediately obtain

$$\frac{r_b}{d} = 2^{\frac{\varkappa^2-3\varkappa+3}{\varkappa(2-\varkappa)}} \varkappa^{\frac{2-\varkappa^2}{2\varkappa(2-\varkappa)}} (\varkappa+1)^{-\frac{\varkappa+1}{2\varkappa}} \left(\varkappa_1 C_{x2}^{\frac{1}{4}}\right)^{\frac{2\varkappa-1}{\varkappa}} \left(\frac{x}{d}\right)^{\frac{1}{2\varkappa}} \tag{4.2}$$

Recalling Eq. (4.1) for the drag coefficient C_{x2}, we see that relation (4.2) becomes (1.15) with $d = r_*$.

We therefore conclude that the methods of asymptotic expansions and deformed coordinates as applied to the solution of the inverse problem of determining the shape of a body from the shock wave (1.13) which it generates, yield the same prescription: the results of intense-explosion theory can be applied without alteration to the entire region between the shock front and the body whose contour is formed by the trajectory of a particle with an entropy corresponding to compression of the gas at the normal shock wave in a steady hypersonic flow.

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DIFFRACTION OF A SHOCK WAVE ON A WEDGE MOVING AT SUPERSONIC SPEED

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We investigate the differentiation of a shock wave of an arbitrary intensity on the upper surface of a wedge moving at supersonic speed under the assumption that the difference between the intensities of the shock wave and the attached shock as well as the difference between the wedge angle α and the angle of incidence of the shock wave δ are both small (Fig. 1).

The case of a flow when a plane shock wave impinges on a wedge moving at supersonic speed and diffraction is absent, was dealt with in [1]. In the present paper we obtain conditions under which a constant parameter flow is realized in the region AFK bounded by the impinging shock wave, the attached shock and the wedge wall.

Diffraction of a shock wave of arbitrary intensity on a slender wedge moving at supersonic speed was dealt with in [2]. Paper [3] was concerned with the diffraction of a weak wave on a slender wedge moving at hypersonic speed. In addition, diffraction of a weak wave on an arbitrary wedge moving at supersonic speed was the theme of a Candidate's